補足 2－6　対数尤度関数(式(2.11))の最大化

対数尤度関数

$$l = \log LF = \sum_{i=1}^{M} \left(G_i \log P_i + U_i \log R_i + L_i \log Q_i \right) \qquad (2.11)$$

の値を最大にするパラメタ μ、σ、c の値を、準ニュートン法（Davidon-Fletcher-Powell 公式）による極値探索法で求める。このときに用いる偏導関数は、次式で与えられるものである。

$$\frac{\partial l}{\partial \mu} = \sum_{i=1}^{M} \left(G_i \cdot \frac{1}{P_i} \cdot \frac{\partial P_i}{\partial \mu} + U_i \cdot \frac{1}{R_i} \cdot \frac{\partial R_i}{\partial \mu} + L_i \cdot \frac{1}{Q_i} \cdot \frac{\partial Q_i}{\partial \mu} \right)$$

$$\frac{\partial l}{\partial \sigma} = \sum_{i=1}^{M} \left(G_i \cdot \frac{1}{P_i} \cdot \frac{\partial P_i}{\partial \sigma} + U_i \cdot \frac{1}{R_i} \cdot \frac{\partial R_i}{\partial \sigma} + L_i \cdot \frac{1}{Q_i} \cdot \frac{\partial Q_i}{\partial \sigma} \right)$$

$$\frac{\partial l}{\partial c} = \sum_{i=1}^{M} \left(G_i \cdot \frac{1}{P_i} \cdot \frac{\partial P_i}{\partial c} + U_i \cdot \frac{1}{R_i} \cdot \frac{\partial R_i}{\partial c} + L_i \cdot \frac{1}{Q_i} \cdot \frac{\partial Q_i}{\partial c} \right)$$

ここで、

$$\frac{\partial P_i}{\partial \mu} = \phi_0\left(\frac{x_i - \mu - c}{\sigma}\right) \cdot \frac{-1}{\sigma} = -\frac{1}{\sigma} \cdot \phi_0\left(\frac{x_i - \mu - c}{\sigma}\right)$$

$$\frac{\partial Q_i}{\partial \mu} = -\phi_0\left(\frac{x_i - \mu + c}{\sigma}\right) \cdot \frac{-1}{\sigma} = \frac{1}{\sigma} \cdot \phi_0\left(\frac{x_i - \mu + c}{\sigma}\right)$$

$$\frac{\partial R_i}{\partial \mu} = -\frac{\partial P_i}{\partial \mu} - \frac{\partial Q_i}{\partial \mu}$$

$$\frac{\partial P_i}{\partial \sigma} = \phi_0\left(\frac{x_i - \mu - c}{\sigma}\right) \cdot (x_i - \mu - c) \cdot \frac{-1}{\sigma^2}$$

$$= \frac{-(x_i - \mu - c)}{\sigma^2} \cdot \phi_0\left(\frac{x_i - \mu - c}{\sigma}\right)$$

$$\frac{\partial Q_i}{\partial \sigma} = -\phi_0\left(\frac{x_i - \mu + c}{\sigma}\right) \cdot (x_i - \mu + c) \cdot \frac{-1}{\sigma^2}$$

$$= \frac{(x_i - \mu + c)}{\sigma^2} \cdot \phi_0\left(\frac{x_i - \mu + c}{\sigma}\right)$$

$$\frac{\partial R_i}{\partial \sigma} = -\frac{\partial P_i}{\partial \sigma} - \frac{\partial Q_i}{\partial \sigma}$$

$$\frac{\partial P_i}{\partial c} = \phi_0\left(\frac{x_i - \mu - c}{\sigma}\right) \cdot \frac{-1}{\sigma} = -\frac{1}{\sigma} \cdot \phi_0\left(\frac{x_i - \mu - c}{\sigma}\right)$$

$$\frac{\partial Q_i}{\partial c} = -\phi_0\left(\frac{x_i - \mu + c}{\sigma}\right) \cdot \frac{1}{\sigma} = -\frac{1}{\sigma} \cdot \phi_0\left(\frac{x_i - \mu + c}{\sigma}\right)$$

$$\frac{\partial R_i}{\partial c} = -\frac{\partial P_i}{\partial c} - \frac{\partial Q_i}{\partial c}$$

であり、$\phi_0(z)$は標準正規分布の確率密度関数を表す。