Y. Okamoto; 2004.2

A modified least squares estimation of ROC line

In general, distance between a line

$$\frac{x}{x_0} + \frac{y}{y_0} = 1 \tag{1}$$

and a point (u, v) is given by

$$\frac{\left|\left(x_0 - u - \frac{x_0}{y_0}v\right)\left(y_0 - v - \frac{y_0}{x_0}u\right)\right|}{\sqrt{\left(x_0 - u - \frac{x_0}{y_0}v\right)^2 + \left(y_0 - v - \frac{y_0}{x_0}u\right)^2}} \tag{2}$$

For the proof, see the appendix.

Program PROCGaussNE.exe estimates the parameters x_0 and y_0 for N data points $(Z(f_i), Z(h_i))$ so that they minimize the sum of squares of distances between the line given by eq. (1) and the data points

$$SS(x_0, y_0) = \sum_{i=1}^{N} \frac{\left(x_0 - Z(f_i) - \frac{x_0}{y_0}Z(h_i)\right)^2 \left(y_0 - Z(h_i) - \frac{y_0}{x_0}Z(f_i)\right)^2}{\left(x_0 - Z(f_i) - \frac{x_0}{y_0}Z(h_i)\right)^2 + \left(y_0 - Z(h_i) - \frac{y_0}{x_0}Z(f_i)\right)^2},$$

This optimization uses Rosenbrock's method (Rao, 1984), which does not need derivatives. From these estimations of x_0 and y_0, the values of μ_s and σ_s are calculated (Wickens, 2002, eq. (3.7)). In the case of equal-variance Gaussian model, estimation is made under the restriction $\sigma_s = \sigma_n = 1$.

When the program starts, a form shown in Fig. 1 appears. Set the values after you prepare the appropriate number of rows in the string grid. Rows can be inserted or deleted by clicking Add or Del buttons. By clicking Add button, you can insert a blank row bellow the one which includes the active cell and, by clicking Del button, you can delete the row with the active cell. A cell becomes active when clicked.

Figure 2 shows the form where the values in Example 3.2 (Wickens, 2002, Pp. 54-55) are set. The values set in the grid can be saved into a CSV file by clicking Save Button.

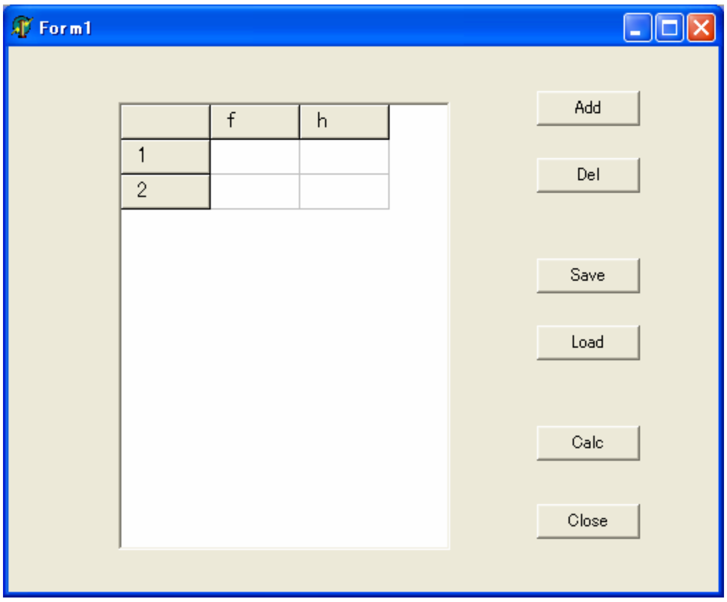


Figure 1.　The form which appears at the start.

Saved data can be loaded by clicking Load button.

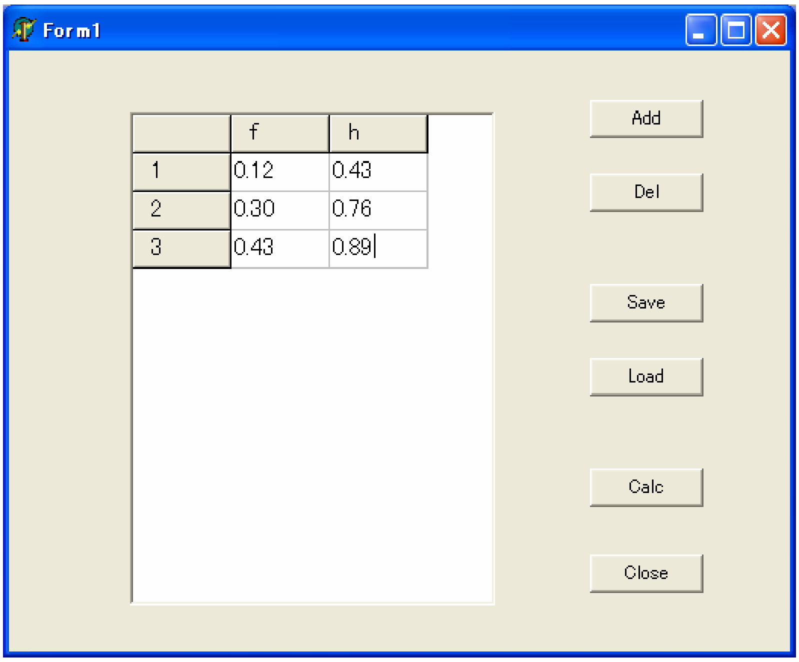


Figure 2.　The form with the values set.

After setting the values, click Calc button, then calculation begins.　The results will be shown as in Figure 3.　The result for equal-variance model is shown in red, and the one for unequal-variance model in black.

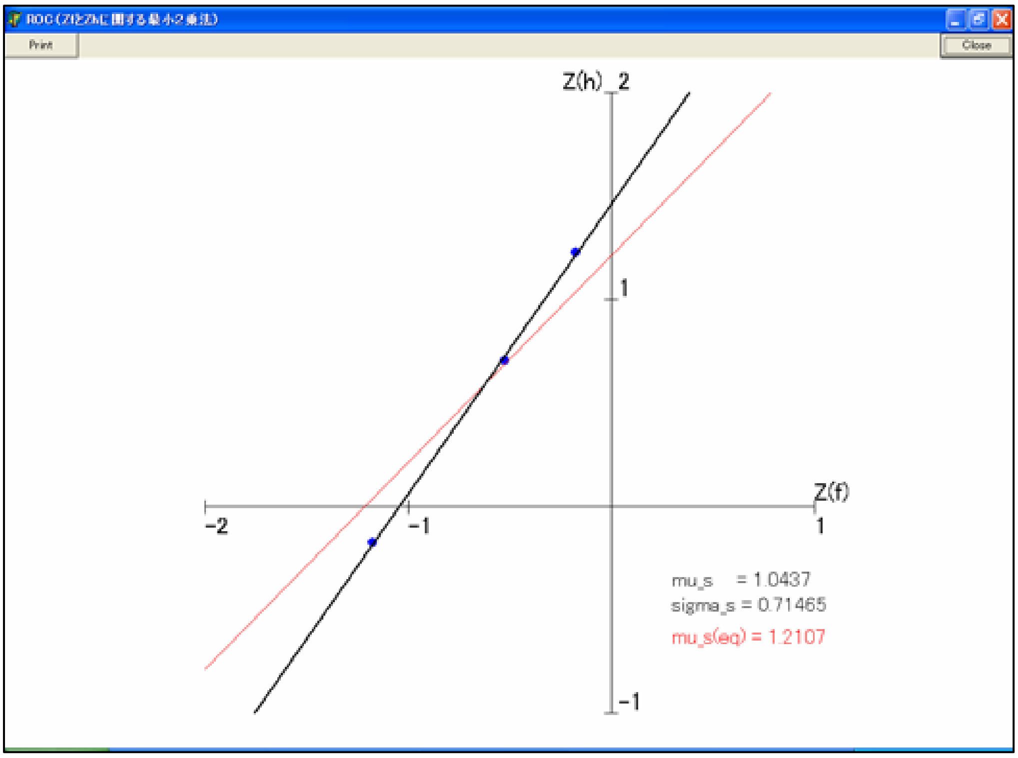


Figure 3 Results of the calculation

mu_s(eq) = 1.2107

means

$$d' = \mu_s = 1.2107$$

for equal-variance model.

mu_s = 1.0437

and

sigma_s = 0.71465

mean that

$$\hat{\mu}_s = 1.0437$$
$$\hat{\sigma}_s = 0.71465$$

for unequal-variance model.

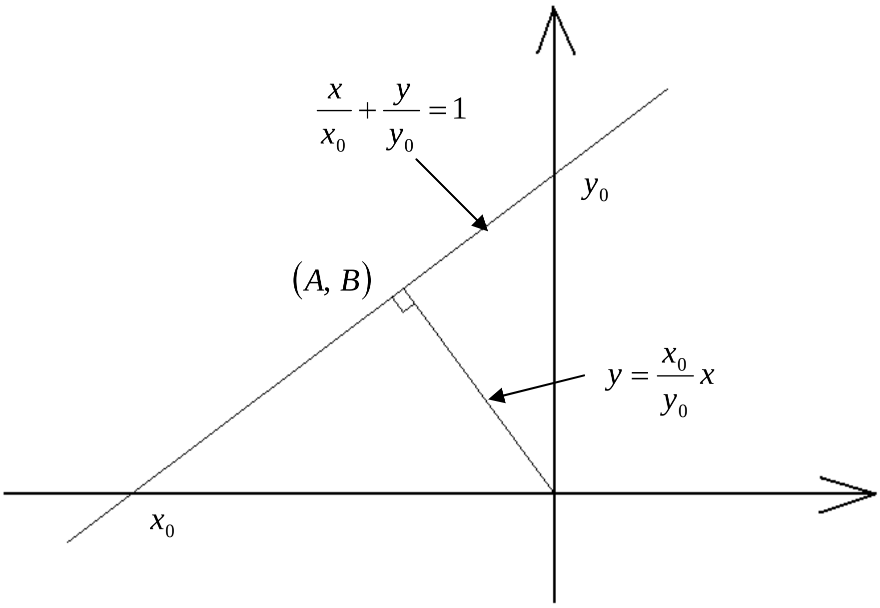
Click Print button to print out the figure.

REFERENCE

Rao, S. S. 1984 *Optimization: Theory and Applications (Second Edition).* John Wiley & Sons.

Appendix

Distance between a line and a point



図ａ1　Distance between a point and a line

First, consider distance between the origin and a line

$$\frac{x}{x_0}+\frac{y}{y_0}=1 \tag{a1}$$

The line which go through the origin and is orthogonal to line (a1) is given by the following equation

$$y=\frac{x_0}{y_0}x \tag{a2}$$

So the intersection (A, B) of lines eq. (a1) and eq. (a2) satisfies the following equation

$$\frac{A}{x_0}+\frac{B}{y_0}=1 \tag{a3}$$

$$B = \frac{x_0}{y_0} A \tag{a4}$$

Replacing B in eq. (a3) by the right hand side of eq. (a4), we get

$$\frac{A}{x_0} + \frac{x_0}{y_0^2} A = 1$$

$$\frac{y_0^2 + x_0^2}{x_0 y_0^2} A = 1$$

$$A = \frac{x_0 y_0^2}{x_0^2 + y_0^2} \tag{a5}$$

Replacing A in eq. (a4) by the right hand side of eq. (a5), we get

$$B = \frac{x_0}{y_0} \cdot \frac{x_0 y_0^2}{x_0^2 + y_0^2}$$

$$= \frac{x_0^2 y_0}{x_0^2 + y_0^2} \tag{a6}$$

Hence the distance between the origin and the line is given by

$$\sqrt{A^2 + B^2} = \sqrt{\left(\frac{x_0 y_0^2}{x_0^2 + y_0^2}\right)^2 + \left(\frac{x_0^2 y_0}{x_0^2 + y_0^2}\right)^2}$$

$$= \sqrt{\frac{x_0^2 y_0^2 \left(y_0^2 + x_0^2\right)}{\left(x_0^2 + y_0^2\right)^2}}$$

$$= \frac{\left|x_0 y_0\right|}{\sqrt{x_0^2 + y_0^2}} \tag{a7}$$

Now, consider distance between point (u, v) and line (a1). Shift x and y axes so that the origin coincides with point (u, v) (Fig. a2). Denote these new axes by X and Y. Relation between the new coordinates and the old ones are given as follows,

$$X + u = x$$

$$Y + v = y$$

Hence, line (a1) is represented by the new coordinate X and Y as follows,

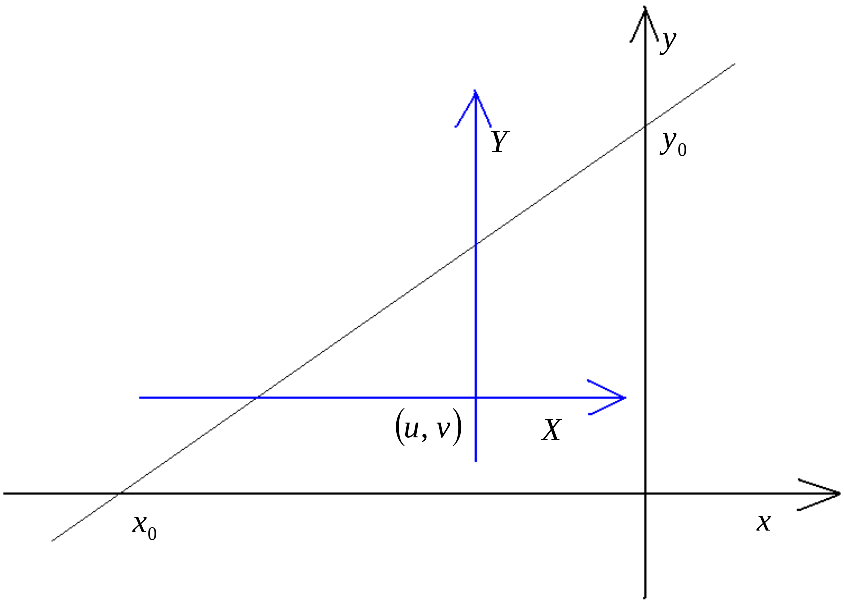


図 a 2　Distance between point (u, v) and the line.

$$\frac{X+u}{x_0}+\frac{Y+v}{y_0}=1$$

$$\frac{X}{x_0}+\frac{Y}{y_0}=1-\frac{u}{x_0}-\frac{v}{y_0}$$

$$\frac{X}{x_0\left(1-\frac{u}{x_0}-\frac{v}{y_0}\right)}+\frac{Y}{y_0\left(1-\frac{u}{x_0}-\frac{v}{y_0}\right)}=1 \qquad \text{(a8)}$$

By eqs. (a7) and (a8), we find that the distance between point (u, v) and line (1) is given by the following equation

$$\frac{\left|\left[x_0\left(1-\frac{u}{x_0}-\frac{v}{y_0}\right)\right]\left[y_0\left(1-\frac{u}{x_0}-\frac{v}{y_0}\right)\right]\right|}{\sqrt{\left[x_0\left(1-\frac{u}{x_0}-\frac{v}{y_0}\right)\right]^2+\left[y_0\left(1-\frac{u}{x_0}-\frac{v}{y_0}\right)\right]^2}}=\frac{\left|\left(x_0-u-\frac{x_0}{y_0}v\right)\left(y_0-v-\frac{y_0}{x_0}u\right)\right|}{\sqrt{\left(x_0-u-\frac{x_0}{y_0}v\right)^2+\left(y_0-v-\frac{y_0}{x_0}u\right)^2}}$$

That is, we get eq. (2).